Math Objectives

- Students will discover how the coefficients of a given trinomial $x^2 + bx + c$ affect the factors.
- Students will discover how the coefficients of a given trinomial $ax^2 + bx + c$ affect the factors.
- Students will reason abstractly and quantitatively (CCSS Mathematical Practice).

Vocabulary

- trinomial
- binomial factors
- algebra tiles

About the Lesson

- This lesson involves factoring trinomials.
- Students will:
 - Edit the text box to select the correct binomial factors of a given trinomial.
 - Reflect on aspects of the "guess-and-check" strategy for factoring trinomials.

TI-Nspire™ Navigator™ System

- Screen Capture and/or Live Presenter may be used to demonstrate the procedure for using the TI-Nspire document file, for monitoring student progress and for displaying instructive examples.
- Quick Poll may be used to assess students' understanding of the concepts throughout or at the end of the lesson.



TI-Nspire[™] Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages

Tech Tips:

 Make sure the font size on your TI-Nspire handheld is set to Medium.

Lesson Materials:

Student Activity

- Factoring_Trinomials_Studen t.pdf
- Factoring_Trinomials_Studen t.doc
- **TI-Nspire document**
- Factoring_Trinomials.tns

Visit <u>www.mathnspired.com</u> for lesson updates and tech tip videos.



Discussion Points and Possible Answers

Tech Tip: It is very important that students select the correct text to edit and not something else. Click on the constant numbers to increase the number. To change from addition to subtraction, click on the sign. Alternatively, students can click on the keypad box in the lower-right corner of the screen. The constant term of the binomial may be changed on page 1.2 while both the coefficient of *x* and the constant term may be changed on page 1.3. The algebra tiles will automatically change to match the binomial factors. Students also will need to use 1^*x as *x*.

Tech Tip: Sometimes while students are working with .tns file, they do something "wrong" that you don't know how to fix or don't have the time to fix. A quick fix is to close the file **without saving** and then reopen the file.

Teacher Tip: The first page deals with monic (a = 1) trinomials. You may want to start these questions together as a class, as a review of factoring from Algebra 1. New trinomials are generated using the minimized slider. You may want students to do more than or less than the eight questions depending upon their expertise.

Move to page 1.2.

The trinomials on this page are in the form: $x^2 + bx + c$. Factor the trinomial given. Change the binomial factors represented by the algebra tiles accordingly. A message will appear when the two binomials are the factors of the trinomial given.



1. Press esc to generate a new trinomial to factor. Fill in the table with eight examples of factored trinomials.

Trinomial	b	С	Factor 1	Factor 2
$x^2 + 9x + 8$	9	8	<i>x</i> + 8	<i>x</i> + 1
$x^2 + x - 12$	1	-12	x – 3	<i>x</i> + 4
$x^2 + 14x + 49$	14	49	<i>x</i> + 7	<i>x</i> + 7
$x^2 - 3x - 40$	-3	-40	<i>x</i> + 5	x – 8
$x^2 - 10x + 21$	-10	21	x – 7	x – 3
$x^2 - 5x - 36$	-5	-36	<i>x</i> – 9	<i>x</i> + 4
$x^2 - 8x - 9$	-8	-9	<i>x</i> + 1	x-9
$x^2 + 4x - 5$	4	-5	<i>x</i> + 5	<i>x</i> – 1

Sample answers: Students will have different trinomials since they are randomly generated.

TI-Nspire Navigator Opportunities

You may use the Navigator to demonstrate the procedure for using the TI-Nspire document file; showing how to edit the text boxes to change the binomials shown. You can also use it to do some of the questions as a class, guessing numbers to try and discussing strategies.

2. Use the examples shown in the table to explain, in your own words, how to factor a trinomial of the form: $x^2 + bx + c$. Be sure to mention the signs in the factors.

<u>Answer:</u> If the trinomial factors over the set of integers, the two binomials will both start with the term x. If the sign of c is positive, the constant terms will both have the same sign (both positive if the sign of b is positive and both negative if the sign of b is negative). The constant terms will have opposite signs if the sign of c is negative. The integers used need to give the value of c when multiplied together and the value of b when added together.

3. Given the trinomial: $x^2 + bx + 12$, find the six possible values of *b* so that the trinomial factors (over the set of integers) to two binomials. State the value of *b* and the factors for each case. Why are these the only factors?

Answer:

b	Factor 1	Factor 2
13	x + 12	<i>x</i> + 1
8	<i>x</i> + 6	x + 2
7	<i>x</i> + 4	x + 3

b	Factor 1	Factor 2
-13	x-12	x – 1
-8	x-6	x-2
-7	x – 4	x-3

When distributing (multiplying) the factors, the numbers must equal the coefficients. In other words, for $x^2 + bx + c$, (x + f) and (x + g) are factors if $f \cdot x + g \cdot x = b \cdot x$ and $f \cdot g = c$.

Move to page 1.3. Read the directions on pages 1.4 and 1.5 to use the simulation on page 1.3.

The trinomials on this page are in the form: $ax^2 + bx + c$.

- 1.1 1.2 1.3 > Factoring_Tri...als ↓ ₩ ₩ 2x - 4 Factor: -2x³ - 20x - 42 3 ★ - 3
- 4. Using trial and error, factor the trinomial given. Fill in the table with eight examples of factored trinomials.

Teacher Tip: Complicated factoring is often difficult for students. They need to find possible factors by making intelligent guesses by looking at the values of *a* and *c*. You may want to do some of these with the entire class, having students make suggestions as to what numbers to try, and discussing strategies for finding the factors.

Sample answer: Students will have different trinomials since they are randomly generated.

Trinomial	а	b	С	Factor 1	Factor 2
$16x^2 + 4x - 12$	16	4	-12	4x + 4	4x - 3
$-6x^2-13x+5$	-6	-13	5	-2x-5	3 <i>x</i> -1
$-3x^2+6x+24$	-3	6	24	-3 <i>x</i> +12	x+2
$2x^2 - 21x + 54$	2	-21	54	x-6	2 <i>x</i> -9
$-6x^2+21x-15$	-6	21	-15	-3x+3	2x - 5
$-4x^2-20x-16$	-4	-20	-16	-4x-4	x+4
$2x^2 + 10x - 72$	2	10	-72	2x - 8	<i>x</i> +9

TI-Nspire Navigator Opportunities

You may monitor the students' progress and stop the class if necessary to discuss some of the conditions on the numbers to try (for example, show a problem where the value of c is negative).

5. Use the examples shown in the table to explain in your own words how to factor a trinomial of the form $ax^2 + bx + c$.

Answer: If the trinomial factors over the set of integers, the two binomials will both start with x terms, and the coefficients must give the value of a when multiplied together. If the sign of c is positive, the constant terms will both have the same sign (both positive if the sign of b is positive and both negative if the sign of b is negative). The constant terms will have opposite signs if the sign of c is negative. The integers used need give the value of c when multiplied together. The set of numbers making the coefficients must each be multiplied by one of the constant terms and then the two products must give the value of b when added together.

6. Given the trinomial: $6x^2 + bx + c$, list all the possible sets of integers that could be the coefficients of x in the binomial factors.

Answer: 1 and 6, 2 and 3, -1 and -6, -2 and -3

Teacher Tip: You might want to ignore the two negative sets above as good mathematical form does not usually allow for negative coefficients for x inside the binomial factors. However, the negative sets might be used in other applications, such as solving an equation. The negative will affect the zero (root) of the equation. This also has to do with the x-intercepts of the graph.

7. Given the trinomial $3x^2 + bx + 2$, find the four possible values of *b* so that the trinomial factors, over the set of integers, to two binomials. State the value of *b* and the factors for each case.

Answer:

b	Factor 1	Factor 2	b	Factor 1	Factor 2
5	3 <i>x</i> +2	<i>x</i> +1	-5	3x-2	x – 1
7	3 <i>x</i> +1	x+2	-7	3 <i>x</i> -1	x – 2

8. For which trinomial would it be easier to find all the factors, $19x^2 + bx - 7$ or $15x^2 + bx - 24$? Explain your answer.

<u>Answer:</u> The first trinomial would be easier to factor, as 19 and 7 are both prime numbers. Their only factors are 1 and themselves. This makes fewer possible combinations of factors to try to get the value of *b* (there are two possible combinations). The second trinomial has two sets of factors for 15 (not allowing negative numbers) and eight sets of factors for –24, making thirty-two possible combinations of factors to try to get the value of *b*. **Teacher Tip:** Students should recognize that if *a* and *c* have many factors, the guess-and-check method is more involved.

9. Drew thinks the trinomial $6x^2 + x - 2$ factors as (2x + 1)(3x - 2), but Kat thinks its factors are (2x + 1)(3x + 2). Who is right? Explain.

Answer: (2x + 1)(3x - 2) expands to $6x^2 - x - 2$ and (2x + 1)(3x + 2) expands to $6x^2 + x - 2$, so Kat is correct.

Teacher Tip: If students didn't notice it in the exercise on page 2.1, now is the time to emphasize the importance of getting the signs correct when the value of *c* is negative. It is also important to mention that not all trinomials can be factored over the integers. One example of a non-factorable trinomial is $3x^2 + 5x + 1$.

Move to pages 2.1–2.2.

Students are to follow the directions on page 2.1 to factor the trinomials at each of the three levels on page 2.2. Level 3 is the most difficult and you may want have students work in pairs to complete this level.



Wrap Up

At the end of the discussion students should be able to:

• Factor trinomials in the form $ax^2 + bx + c$ over the integers by making educated guesses using combinations of the factors of *a* and *c*.

Assessment

Use Quick Poll and the following questions to assess students' understanding of this concept.

Sample Questions:

- 1. The factors of the trinomial $x^2 + bx 11$ are:
 - a. (x+11) and (x+1) if the value of b is positive
 - b. (x-11) and (x-1) if the value of b is positive

C. (x+11) and (x-1) if the value of b is positive



d. (x-11) and (x+1) if the value of b is positive



- 2. The factors of the trinomial $2x^2 + bx + 1$ are:
 - a. (2x+1) and (x+1) if the value of b is negative
 - b. (2x-1) and (x-1) if the value of b is negative
 - c. (2x+1) and (x-1) if the value of b is negative
 - d. (2x-1) and (x+1) if the value of b is negative
- 3. The factors of the trinomial $3x^2 + bx 5$ are:
 - a. (3x+5) and (x+1) if the value of *b* is positive
 - b. (3x-5) and (x-1) if the value of *b* is positive
- C. (3x+5) and (x-1) if the value of b is positive
 - d. (3x-5) and (x+1) if the value of b is positive