Asymptotes and Zeros

About the Lesson

In this activity, students relate the graph of a rational function to the graphs of the polynomial functions of its numerator and denominator. Students graph these polynomials one at a time and identify their y-intercepts and zeros. These features of the graph are connected with the standard and factored forms of the equation. Using the handheld's manual manipulation functions, students can manipulate the graphs of the numerator and denominator functions and see the effect on the rational function. As a result, students will:

- Graph any rational function and identify its singularities and asymptotes.
- Factor the denominator of a rational function to locate its singularities.
- Approximate the solutions to rational equations and inequalities by graphing.

Vocabulary

- polynomial function
- rational function
- asymptote
- zero

Teacher Preparation and Notes

- This activity is designed to be used in an Algebra 2 or Precalculus classroom.
- This activity is intended to be mainly **teacher-led** with brief periods of independents student work.
- This worksheet helps guide students through the activity and provides a place for them to record their answers.
- Prior to beginning this activity, students should have an introduction to polynomial functions, their graphs, and the concept of degree. This activity may serve as an introduction to rational functions.

Activity Materials

- Compatible TI Technologies:

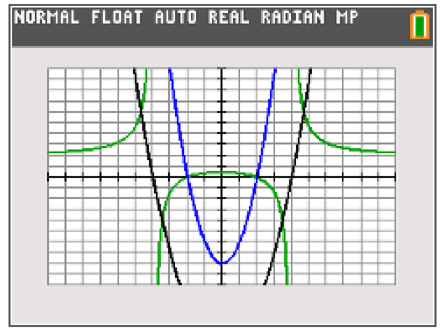
  TI-84 Plus*

  TI-84 Plus Silver Edition*

  TI-84 Plus C Silver Edition

  TI-84 Plus CE

*** *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



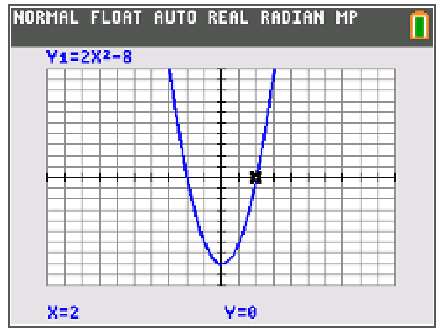
Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

Lesson Files:

- Asymptotes_and_Zeros_Student.pdf
- Asymptotes_and_Zeros_Student.doc

Students are introduced to the definition of a rational function. They will first graph the numerator of $y = \dfrac{2x^2 - 8}{x^2 - 16}$ and explore its *y*-intercept and zeros using the trace feature.



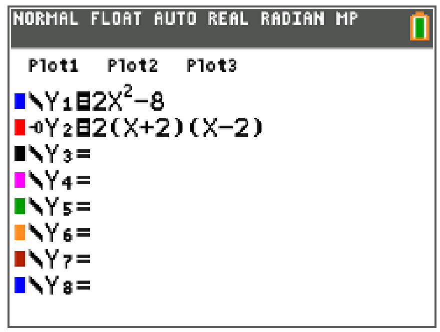
1. Where are the zeros?

   Answer: The zeros are at (–2, 0) and (2, 0).

2. What is the *y*-intercept?

   Answer: The *y*-intercept is at (0, –8).

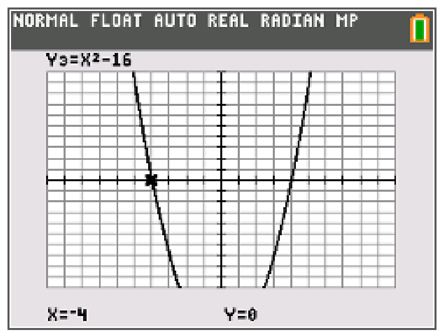
Using the zeros found, students should write the factored form of
$y = 2x^2 - 8$ and enter it in **Y2**. Make sure that they change the attribute of the graph to **–8** before pressing $\boxed{\text{graph}}$.



If they enter $y = 2(x + 2)(x - 2)$, then the circle should trace the original graph.

Now students are to turn off **Y1** and **Y2** and enter the denominator of $y = \dfrac{2x^2 - 8}{x^2 - 16}$ into **Y3**.

They should find that the *y*-intercept is at (0, –16) and the zeros are at (–4, 0) and (4, 0).

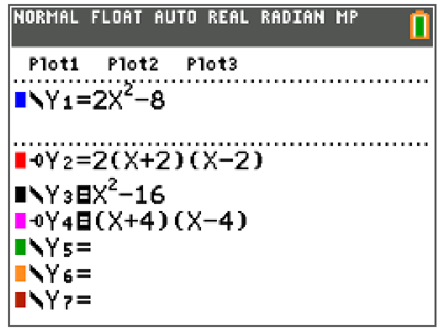


3. How does the factored form relate to the zeros of the function?

   Answer: The zeros of the function occur at the values of *x* where each factor equals zero.

Using the zeros found, students should write the factored form of $y = x^2 - 16$ and enter it in **Y4**. Make sure that they change the attribute of the graph to **–0** before pressing $\boxed{\text{graph}}$.

If they enter $y = (x + 4)(x - 4)$, then the circle should trace the original graph.



4. What indication did the equation give of what the y-intercept would be? Record the value.

   The y- intercept for *denominator* is y = _____________.

   Answer: The value of the constant indicates the y-intercept. The y-intercept is –16.

5. Continue to trace to find the zeros. Record.

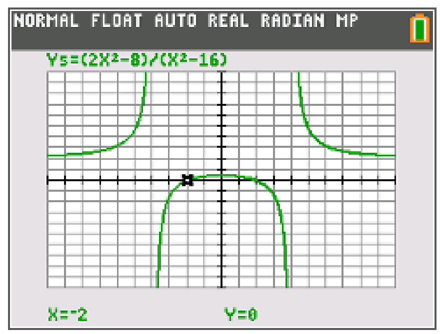
   The *denominator* is zero at x = ________________.

   Answer: The denominator is zero at x = –4 and 4.

6. How does the factored form relate to the zeros of the function?

   Answer: The zeros of the function occur at the values of x where each factor equals zero.

Now students are to turn off **Y3** and **Y4** off and enter the entire rational function $y = \dfrac{2x^2 - 8}{x^2 - 16}$ into **Y5**.



They should see that the zeros are located at the same x-values that the zeros of the numerator are located and the vertical asymptotes occur at the same location as the zeros of the denominator.

Discuss with students that vertical asymptotes occur when the denominator of the function is equal to 0, which makes the function undefined.

7. On cursory inspection, where do you see interesting things happening on this graph?

   Sample Answer: Students will most likely describe the “breaks” in the graph, which are the asymptotes. They may also notice the places where the graph crosses the axes.

8. Is this a graph of a function?

   Answer: Yes. (The graph passes the vertical line test.)

9. For what *x*-values do the zeros of this function occur?

   Answer: The zeros are at (–2, 0) and (2, 0).

10. The zeros of this function occur at the same locations as the zeros of the numerator. Why is this true?

    Answer: The zeros of the graph only depend on the numerator of the rational function

11. There appear to be some vertical lines on the graph where a break in the graph occurs. Where do these appear?

    Answer: The vertical lines occur at $x = -4$ and 4.

12. While still in trace mode, type in 4 for $x = 4$. What is the *y*-value at this point? At $x = -4$?

    Answer: There is no *y*-value because the function is not defined at either $x = -4$ or $x = 4$.

13. What happens when the denominator of a fraction is zero?

    Answer: The value of the fraction is undefined.

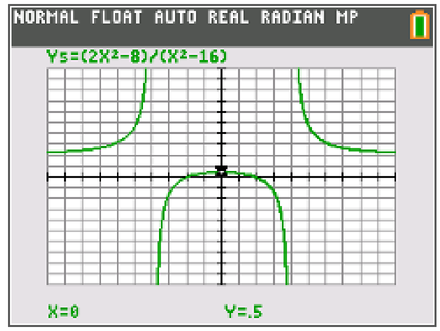
14. Lastly, look for the *y*-intercept. What is the value of *y* when *x* is 0?

    Answer: The value of *y* is $\frac{1}{2}$.

15. Recall that the *y*-intercept of the numerator is –8 and the *y*-intercept of the denominator is –16. What is the quotient of these two values?

    Answer: $\frac{-8}{-16} = \frac{1}{2}$ or 0.5

The graph of the original rational function $y=\dfrac{2x^2-8}{x^2-16}$ is shown to the right.



If students graph the functions of the numerator, denominator, and rational function at the same time, they will see that the parabola of the numerator intersect at the rational function's zeros and the parabola of the denominator intersects at the rational function's vertical asymptotes.